2.227. $u(x,t) = \dfrac{2l_0u_0}{\pi(l_0-x)}\sum_{n=1}^{\infty}\dfrac{1}{n}\sin\dfrac{\pi nx}{l}e^{-\left(\frac{\pi an}{l}\right)^2 t}$.

2.228. $$u(r,t) = \frac{9Q}{56\pi D} + \frac{Q}{2\pi D}\sum_{n=1}^{\infty}\frac{4\mu_n^2+1}{\mu_n^3(4\mu_n^2+7)}\left(1+(-1)^{n+1}\sqrt{\frac{\mu_n^2+1}{4\mu_n^2+1}}\right)\times$$
$$\times R_n(r)e^{(a\mu_n)^2 t},\ \ R_n(r) = \frac{1}{r}(\sin\mu_n(r-1)+\mu_n\cos\mu_n(r-1)),$$
$\mu_n>0$ — корень уравнения $\mu\cos\mu = (1+2\mu^2)\sin\mu$.

Указание. См. задачи 2.20, п. 4.

2.229. 1) $$u(r,t) = \frac{8A(r_2-r_1)^2}{\pi^3 r}\sum_{n=1}^{\infty}\frac{\sin\dfrac{\pi(2n+1)(r-r_1)}{r_2-r_1}\cos\dfrac{\pi a(2n+1)t}{r_2-r_1}}{(2n+1)^3};$$
2) $$u(r,t)=\frac{4A(r_2-r_1)^2}{\pi^3 r}\sum_{n=1}^{\infty}\frac{(2+(-1)^n)(r_1-(-1)^n)}{n^4}\sin\frac{\pi n(r-r_1)}{r_2-r_1}\cos\frac{\pi ant}{r_2-r_1}t.$$

2.230. $$u(r,t)=\frac{4A(r_2-r_1)^3}{ar}\sum_{n=1}^{\infty}\frac{\mu_n^2+p^2}{\mu_n^4(\mu_n^2+p^2-p)}\left(1+\frac{(-1)^n p}{\sqrt{\mu_n^2+p^2}}\right)\sin\frac{\mu_n(r-r_1)}{r_2-r_1}\times$$
$$\times\sin\frac{a\mu_n t}{r_2-r_1},\ \ \mu_n>0 \text{ — корень уравнения } \mu\cos\mu = p\sin\mu,\ p=1-\frac{r_1}{r_2}.$$

2.231. $$u(x,t) = -\frac{2(l_0-l)I}{\rho_0 aS_2(l_0-x)}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{\mu_n^2+p^2}}{\mu_n^2+p^2+p}\sin\frac{\mu_n x}{l}\ \sin\frac{\mu_n at}{l},$$
$\mu_n>0$ — корень уравнения $\mu\ \mathrm{ctg}\ \mu = -p$, $p = \dfrac{l}{l_0-l}$, $l_0 = \dfrac{\sqrt{S_1}}{\sqrt{S_1}-\sqrt{S_2}}l$.

2.232. $$u(x,t) = \frac{I}{m_0}\left(t+\frac{2l^2}{3a(l-x)}\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{1+\mu_n^2}}{\mu_n^2}\sin\mu_n\left(1-\frac{x}{l}\right)\ \sin\ \frac{\mu_n a}{l}t\right),$$
$\mu_n>0$ — корень уравнения $\mathrm{tg}\ \mu=\mu$, m_0 — масса стержня.

2.233. $$u(x,y) = \frac{4u_1}{\pi}\sum_{n=1}^{\infty}\frac{\mathrm{sh}\,\dfrac{(2n+1)\pi x}{l_2}\ \sin\dfrac{(2n+1)\pi y}{l_2}}{(2n+1)\,\mathrm{sh}\,\dfrac{(2n+1)\pi l_1}{l_2}} +$$
$$+\frac{4u_2}{\pi}\sum_{n=1}^{\infty}\frac{\mathrm{sh}\,\dfrac{(2n+1)\pi y}{l_1}\ \sin\dfrac{(2n+1)\pi x}{l_1}}{(2n+1)\,\mathrm{sh}\,\dfrac{(2n+1)\pi l_2}{l_1}}.$$

Указание. Решение представить в виде суммы $u(x,y) = u_1(x,y)+u_2(x,y)$, где $u_1(x,y)$ удовлетворяет однородным граничным условиям при $y=0$ и $y=l_2$, а $u_2(x,y)$ — однородным граничным условиям при $x=0$ и $x=l_1$.

2.234. $$u(x,y) = \frac{12u_0}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\,\mathrm{ch}\,\dfrac{\pi n(l_1-x)}{l_2}\ \sin\dfrac{\pi ny}{l_2}}{n^3\,\mathrm{ch}\,\dfrac{\pi nl_1}{l_2}}.$$

2.235. 1) $u(x,y) = \dfrac{q_0 l}{2\pi k}\sin\dfrac{2\pi y}{l}e^{-\frac{2\pi x}{l}}$;

2) $$u(x,y) = \frac{8q_0 l}{\pi^2 k}\sum_{n=0}^{\infty}\frac{(-1)^n\cos\dfrac{(2n+1)\pi y}{2l}}{(2n+1)^2}e^{-\frac{(2n+1)\pi x}{2l}}.$$

2.236. $$u(x,y) = \frac{Q}{4\pi kh}\ln\frac{\mathrm{ch}\,\dfrac{\pi x}{2l}+\cos\dfrac{\pi y}{2l}}{\mathrm{ch}\,\dfrac{\pi x}{2l}-\cos\dfrac{\pi y}{2l}}.$$