

где $f(x) = \frac{Q_0}{lD} \cdot \begin{cases} (l-x_0)x, & 0 \leq x \leq x_0, \\ (l-x)x_0, & x_0 \leq x \leq l; \end{cases}$

если $\alpha > \beta$, $\frac{\alpha - \beta}{D} = \gamma^2 \neq \lambda_n$, то

$$u(x, t) = f(x)e^{-\alpha t} - \frac{2Q_0e^{-\beta t}}{lD} \sum_{n=1}^{\infty} \frac{X_n(x_0)X_n(x)e^{-D\lambda_n t}}{\lambda_n - \gamma^2},$$

где $f(x) = \frac{Q_0}{D\gamma \sin \gamma l} \cdot \begin{cases} \sin \gamma(l-x_0) \sin \gamma x, & 0 \leq x \leq x_0, \\ \sin \gamma(l-x) \sin \gamma x_0, & x_0 \leq x \leq l; \end{cases}$

если $\alpha > \beta$, $\frac{\alpha - \beta}{D} = \lambda_{n_0}$, $X_{n_0}(x_0) = 0$, то $x_0 = \frac{ml}{n_0}$, $0 < m < n_0$,

$$u(x, t) = f(x)e^{-\alpha t} - \frac{2lQ_0e^{-\beta t}}{\pi^2 D} \sum_{\substack{n=1 \\ n \neq n_0}}^{\infty} \frac{X_n(x_0)X_n(x)e^{-D\lambda_n t}}{n^2 - n_0^2},$$

где $f(x) = (-1)^m \frac{Q_0}{\pi D n_0} \sin \frac{\pi n_0 x}{l} \cdot \begin{cases} l-x_0, & 0 \leq x \leq x_0, \\ -x_0, & x_0 \leq x \leq l. \end{cases}$ m — целое число;

если $\alpha > \beta$, $\frac{\alpha - \beta}{D} = \lambda_{n_0}$, $X_{n_0}(x_0) \neq 0$, то

$$u(x, t) = f(x)e^{-\alpha t} + \frac{2Q_0}{l} \sin \frac{\pi n_0 x_0}{l} \sin \frac{\pi n_0 x}{l} te^{-\alpha t} - \frac{2lQ_0e^{-\beta t}}{\pi^2 D} \sum_{\substack{n=1 \\ n \neq n_0}}^{\infty} \frac{X_n(x_0)X_n(x)e^{-D\lambda_n t}}{n^2 - n_0^2},$$

где функция

$$f(x) = \frac{Q_0}{\pi n_0 D} \left\{ \left[\frac{l}{2\pi n_0} \sin \frac{\pi n_0 x_0}{l} + (l-x_0) \cos \frac{\pi n_0 x_0}{l} \right] \sin \frac{\pi n_0 x}{l} - x \sin \frac{\pi n_0 x_0}{l} \cos \frac{\pi n_0 x}{l} \right\}$$

определена при $0 \leq x \leq x_0$, а при $x_0 \leq x \leq l$ выражение для $f(x)$ получается из данного заменами $x \rightarrow x_0$, $x_0 \rightarrow x$.

$$2.421. u(x, t) = \frac{Al^2}{a^2} \left(\frac{a^2 xt}{l^3} + \frac{x^3}{6l^3} - \frac{x}{6l} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \sin \frac{n\pi x}{l} e^{-\left(\frac{n\pi a}{l}\right)^2 t}}{n^3} \right).$$

$$2.422. u(x, t) = \frac{Al^3}{ka^2} \left(\frac{a^2 xt}{l^3} + \frac{x^3}{6l^3} - \frac{x}{2l} + \frac{32}{\pi^4} \sum_{n=0}^{\infty} \frac{(-1)^n \sin \frac{(2n+1)\pi x}{2l} e^{-\left(\frac{(2n+1)\pi a}{2l}\right)^2 t}}{(2n+1)^4} \right).$$

$$2.423. u(x, t) = \frac{Al^2}{D} \left(\frac{Dt}{l^2} - \frac{1}{2} + \frac{x^2}{2l^2} + \frac{16}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n \sin \frac{(2n+1)\pi x}{2l} e^{-\left(\frac{(2n+1)\pi a}{2l}\right)^2 t}}{(2n+1)^3} \right).$$

$$2.424. u(x, t) = \frac{Al^3}{D^2} \left[\frac{Dt}{2l^2} \left(\frac{x^2}{l^2} + \frac{Dt}{l^3} - \frac{1}{3} \right) - \frac{x^2}{12l^2} \left(1 - \frac{x^2}{2l^2} \right) + \frac{7}{360} + \frac{2}{\pi^4} \sum_{n=1}^{\infty} \frac{(-1)^n \cos \frac{n\pi x}{l} e^{-\left(\frac{n\pi a}{l}\right)^2 t}}{n^4} \right].$$

$$2.425. u(x, t) = \frac{Al^2}{D} \left(\frac{Dt}{l^2} + \frac{x^2}{2l^2} - \frac{p+2}{2p} - 2p \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{\mu_n^2 + p^2} \cos \frac{\mu_n x}{l} e^{-\left(\frac{\mu_n a}{l}\right)^2 t}}{\mu_n^2 (\mu_n^2 + p^2)} \right),$$

$p = hl$, $h = \frac{\alpha}{D}$, $\mu_n > 0$ — корень уравнения $\mu \operatorname{tg} \mu = p$.